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What We Don't Know Doesn't Hurt Us: Rational Inattention and the Permanent Income Hypothesis in General Equilibrium^{*}

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Abstract

This paper derives the general equilibrium effects of rational inattention (or RI; Sims 2003, 2010) in a model of incomplete income insurance (Huggett 1993, Wang 2003). We show that, under the assumption of CARA utility with Gaussian shocks, the Permanent Income Hypothesis (PIH) arises in equilibrium, as in models with full information-rational expectations, due to a balancing of precautionary savings and impatience. We then explore how RI affects the equilibrium joint dynamics of consumption, income and wealth, and find that elastic attention can make the model fit the data better. We finally show that the welfare costs of incomplete information are even smaller due to general equilibrium adjustments in interest rates.

Keywords: Rational Inattention; Permanent Income Hypothesis; General Equilibrium; Consumption and Income Volatility.

JEL Classification Numbers: C61; E21; D83.

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1. Introduction

In intertemporal consumption-savings problems, households save today for three reasons: (i) they anticipate future declines in income, (ii) they face uninsurable risk that generates precautionary savings, and (iii) they are relatively patient compared to the interest rate. When only motive (i) is operative then one obtains the “permanent income hypothesis (PIH)” of Friedman (1957), where consumption is solely determined by permanent income and follows a random walk. The PIH has some implications that are strongly inconsistent with the data. Two implications in particular are discussed in Campbell and Deaton (1989), the excess sensitivity and excess smoothness puzzles. Excess sensitivity occurs if consumption responds to predictable changes in income; under the PIH those changes are part of permanent income and therefore have already had their effect on consumption. Excess smoothness occurs if consumption responds less than one for one to permanent changes in income (or equivalently less than one for one to changes in permanent income). The two puzzles are actually manifestations of the same underlying economic forces, as shown in Campbell and Deaton (1989), and their absence is profoundly rejected. Unfortunately, uninsurable income risk seems to be pervasive in microeconomic data, and general equilibrium models with uninsurable risk tend to predict impatience of households (that is, they face “low interest rates”), so that the basic PIH is violated.

Wang (2003), using a simple model with CARA utility and risk free assets in zero net supply, shows that the PIH reemerges in general equilibrium – when decision rules are linear, the equilibrium interest rate exactly balances the forces of precautionary saving and dissaving due to impatience, even in the presence of uninsurable risk. Due to the linearity of consumption as a function of individual permanent income, Wang (2003) is able to analytically characterize the forces that operate in general equilibrium and show they cancel out, under some mild assumptions about the labor income process.

Luo (2008) and Luo and Young (2010) introduce rational inattention into the basic partial equilibrium PIH environment; RI implies that agents process signals slowly and therefore appear to respond sluggishly to innovations in permanent income. This sluggish response appears to deliver changes in consumption in response to anticipated income changes, because econometricians actually observe more than the agents do, and as a result also delivers smaller responses to permanent income changes.

Our goal in this paper is to explore the general equilibrium implications of rational inattention in

a model with precautionary savings and ask the same question from Wang (2003) – namely, does the PIH reemerge in general equilibrium – in the presence of rational inattention. We study economies with constant absolute risk aversion (CARA) preferences, as they simultaneously generate precautionary savings and linear consumption rules, and characterize the forces that act on the general equilibrium interest rate. We find that the PIH does describe equilibrium consumption behavior in general equilibrium, with the appropriate substitution of actual permanent income by perceived permanent income. Thus, the delicate canceling of precautionary and impatience forces found by Wang (2003) carries over unmodified to models with incomplete information about the state.¹

One key result in this paper is that there exists general equilibrium interest rates clearing the asset market and they are significantly affected by the degree of RI. After obtaining the explicit expression for consumption dynamics, we examine how RI affects the stochastic properties of the joint dynamics of consumption growth to income growth in both the fixed capacity and elastic capacity cases. Specifically, we find that the effect of RI on consumption dynamics is attenuated by general equilibrium adjustment in the interest rate – as processing capacity declines the interest rate also declines, leading to lower consumption volatility. The implication is that the costs of incomplete information have likely been overestimated in the literature, despite being very tiny to start.² Finally, we compare rational inattention with habit formation, and show that although both hypotheses lead to slow adjustment in consumption, they have opposite effects on the equilibrium interest rate and the relative volatility of consumption growth to income growth.

This paper is organized as follows. Section 2 constructs a precautionary saving model with a continuum of inattentive consumers who have the CARA utility and face uninsurable labor income. Section 3 solves optimal consumption-saving rules under rational inattention and establishes the general equilibrium of this economy. Section 4 examines how RI affects the interest rate and the joint dynamics of consumption, income, and wealth quantitatively. Section 5 concludes.

¹Luo and Young (2014) document an observational equivalence between rational inattention and signal extraction in linear-quadratic-Gaussian models.

²For the welfare losses due to imperfect information about current income or permanent income calculated in the partial equilibrium linear-quadratic (LQ) permanent income models, see Pischke (1995), Luo and Young (2010), and Luo, Nie, and Young (2014). Maćkowiak and Wiederholt (2013) also find tiny welfare losses due to RI over business cycles in a general equilibrium model.

2. A Caballero-Huggett-Wang Economy with Rational Inattention

2.1. A Full-information Rational Expectations Model with Precautionary Savings

Following Caballero (1990) and Wang (2003), we formulate a full-information rational expectations (FI-RE) model with precautionary savings as follows:

$$\max_{\{c_t\}} U(c) = E_0 \left[\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t u(c_t) \right], \quad (1)$$

subject to the flow budget constraint

$$a_{t+1} = (1+r)a_t + y_t - c_t, \quad (2)$$

where $u(c_t) = -\exp(-\alpha c) / \alpha$ is a constant-absolute-risk-aversion utility with $\alpha > 0$, $\rho > 0$ is the agent's subjective discount rate, r is a constant rate of interest, and labor income, y_t , follows a stationary AR(1) process with Gaussian innovations:

$$y_t = \phi_0 + \phi_1 y_{t-1} + w_t, \quad t \geq 1, \quad |\phi_1| < 1, \quad (3)$$

where $w_t \sim N(0, \sigma^2)$, $\phi_0 = (1 - \phi_1) \bar{y}$, \bar{y} is the mean of y_t , and the initial levels of labor income y_0 and asset a_0 are given.³ Solving (1) subject to (2) and (3) yields the following optimal consumption plan:

$$c_t = r \left\{ a_t + h_t + \frac{1}{\alpha r^2} \left[\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t [\exp(-r\alpha\phi w_{t+1})] \right] \right\}, \quad (4)$$

where

$$h_t \equiv \frac{1}{1+r} E_t \left[\sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j y_{t+j} \right], \quad (5)$$

is human wealth defined as the discounted expected present value of current and future labor income and is equal $h_t = \phi(y_t + \phi_0/r)$ after substituting (3) into (5), and $\phi = 1/(1+r-\phi_1)$.⁴ This consumption function is the same as that obtained in Wang (2003). In the last two terms in (4), $\ln \left(\frac{1+\rho}{1+r} \right) / r\alpha$ measures the relative importance of impatience and the interest rate in determining current consumption, and $\ln E_t [\exp(-r\alpha\phi w_{t+1})] / r\alpha$ measures the amount of precautionary sav-

³It is worth noting that assuming that the individual income shock includes two components, one is permanent and the other is transitory, does not change the main results in this paper. Here we follow Wang (2003) to adopt specification (3). The detailed derivation of the model with the two-income shock specification is available from the corresponding author by request. For the empirical studies on the income specification, see Attanasio and Pavoni (2007).

⁴See Appendix 6.1 for the derivation.

ings determined by the interaction of risk aversion and income uncertainty.

In order to facilitate the introduction of rational inattention we follow Luo (2008) and Luo and Young (2010) and reduce the multivariate model to a univariate model with iid innovations to permanent income. Letting permanent income, $s_t = a_t + h_t$, be defined as a new state variable, we can reformulate the PIH model as

$$v(s_t) = \max_{c_t} \left\{ u(c_t) + \frac{1}{1+\rho} E_t[v(s_{t+1})] \right\},$$

subject to

$$s_{t+1} = (1+r)s_t - c_t + \zeta_{t+1}, \quad (6)$$

where the time $(t+1)$ innovation to permanent income can be written as

$$\zeta_{t+1} \equiv \frac{1}{1+r} \sum_{j=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{j-(t+1)} (E_{t+1} - E_t)[y_j], \quad (7)$$

which can be reduced to $\zeta_{t+1} = \phi w_{t+1}$ when we use the income specification, (3), where $v(s_t)$ is the consumer's value function under FI-RE.⁵

2.2. Incorporating Rational Inattention

In this section, we follow Sims (2003) and incorporate rational inattention (RI) due to finite information-processing capacity into the above permanent income model with the CARA-Gaussian specification. Under RI, consumers have only finite Shannon channel capacity to observe the state of the world. Specifically, we use the concept of entropy from information theory to characterize the uncertainty about a random variable; the reduction in entropy is thus a natural measure of information flow.⁶ With finite capacity $\kappa \in (0, \infty)$, a random variable $\{s_t\}$ following a continuous distribution cannot be observed without error and thus the information set at time $t+1$, denoted \mathcal{I}_{t+1} , is generated by the entire history of noisy signals $\{s_j^*\}_{j=0}^{t+1}$. Following the literature, we assume the noisy signal takes the additive form

$$s_{t+1}^* = s_{t+1} + \tilde{\zeta}_{t+1},$$

⁵See Appendix 6.1 for the derivation.

⁶Formally, entropy is defined as the expectation of the negative of the (natural) log of the density function, $-E[\ln(f(s))]$. For example, the entropy of a discrete distribution with equal weight on two points is simply $E[\ln_2(f(s))] = -0.5 \ln(0.5) - 0.5 \ln(0.5) = 0.69$, and the unit of information contained in this distribution is 0.69 "nats". (For alternative bases for the logarithm, the unit of information differs; with log base 2 the unit of information is the 'bit' and with base 10 it is a 'dit' or a 'hartley'.) In this case, an agent can remove all uncertainty about s if the capacity devoted to monitoring s is $\kappa = 0.69$ nats.

where ζ_{t+1} is the endogenous noise caused by finite capacity.⁷ We further assume that ζ_{t+1} is an iid idiosyncratic shock and is independent of the fundamental shocks hitting the economy. The reason that the RI-induced noise is idiosyncratic is that the endogenous noise arises from the consumer's own internal information-processing constraint. Agents with finite capacity will choose a new signal $s_{t+1}^* \in \mathcal{I}_{t+1} = \{s_1^*, s_2^*, \dots, s_{t+1}^*\}$ that reduces the uncertainty about the variable s_{t+1} as much as possible. Formally, this idea can be described by the information constraint

$$\mathcal{H}(s_{t+1}|\mathcal{I}_t) - \mathcal{H}(s_{t+1}|\mathcal{I}_{t+1}) \leq \kappa, \quad (8)$$

where κ is the investor's information channel capacity, $\mathcal{H}(s_{t+1}|\mathcal{I}_t)$ denotes the entropy of the state prior to observing the new signal at $t+1$, and $\mathcal{H}(s_{t+1}|\mathcal{I}_{t+1})$ is the entropy after observing the new signal. κ imposes an upper bound on the amount of information flow – that is, the change in the entropy – that can be transmitted in any given period. Finally, following the literature, we suppose that the prior distribution of s_{t+1} is Gaussian.

Although we adopt the CARA-Gaussian setting in our model, we will assume the loss function due to imperfect-state-observation is still quadratic. Using a quadratic loss function, Sims (2003) shows that the true state under RI also follows a normal distribution $s_t|\mathcal{I}_t \sim N(E[s_t|\mathcal{I}_t], \Sigma_t)$, where $\Sigma_t = E_t[(s_t - \hat{s}_t)^2]$. In addition, given that the noisy signal takes the additive form $s_{t+1}^* = s_{t+1} + \zeta_{t+1}$, the noise ζ_{t+1} will also be Gaussian. In this case, (8) reduces to

$$\ln(|\Psi_t|) - \ln(|\Sigma_{t+1}|) \leq 2\kappa, \quad (9)$$

where $\Sigma_{t+1} = \text{var}_{t+1}(s_{t+1})$ and $\Psi_t = \text{var}_t(s_{t+1}) = (1+r)^2 \Sigma_t + \text{var}_t(\zeta_{t+1})$ are the posterior and prior variance of the state variable, s_{t+1} , respectively. In our univariate model, (9) fully determines the value of the steady state conditional variance Σ :

$$\Sigma = \frac{\text{var}_t(\zeta_{t+1})}{\exp(2\kappa) - (1+r)^2}, \quad (10)$$

which means that Σ is entirely determined by the variance of the exogenous shock ($\text{var}_t(\zeta_{t+1})$) and finite capacity (κ).⁸ Following the steps outlined in Luo and Young (2013), we can compute the

⁷For other types of imperfect information about state variables, see Pischke (1995) and Wang (2004). Pischke (1995) assumes that consumers ignore the aggregate income component, and Wang (2004) assume that consumers cannot distinguish two individual components in the income process.

⁸Note that here we need to impose the restriction $\exp(2\kappa) - (1+r)^2 > 0$. If this condition fails, the state is not stabilizable and the unconditional variance diverges.

Kalman gain in the steady state θ as

$$\theta = 1 - 1/\exp(2\kappa); \quad (11)$$

θ measures the fraction of uncertainty removed by a new signal in each period.⁹

The evolution of the estimated state \hat{s}_t is governed by the Kalman filtering equation

$$\hat{s}_{t+1} = (1 - \theta) ((1 + r) \hat{s}_t - c_t) + \theta s_{t+1}^*. \quad (12)$$

Combining (6) with (12) yields

$$\hat{s}_{t+1} = (1 + r) \hat{s}_t - c_t + \hat{\zeta}_{t+1}, \quad (13)$$

where

$$\hat{\zeta}_{t+1} = \theta (1 + r) (s_t - \hat{s}_t) + \theta (\zeta_{t+1} + \xi_{t+1}) \quad (14)$$

is the innovation to \hat{s}_{t+1} and is independent of all the other terms on the RHS of (13). $\hat{\zeta}_{t+1}$ is an MA(∞) process with $E_t [\hat{\zeta}_{t+1}] = 0$ and

$$\text{var}(\hat{\zeta}_{t+1}) = \Gamma(\theta, r) \omega_{\zeta}^2,$$

where $\Gamma(\theta, r) = \frac{\theta}{1 - (1 - \theta)(1 + r)^2} > 1$ for $\theta < 1$, and

$$s_t - \hat{s}_t = \frac{(1 - \theta) \zeta_t}{1 - (1 - \theta)(1 + r) \cdot L} - \frac{\theta \xi_t}{1 - (1 - \theta)(1 + r) \cdot L} \quad (15)$$

is the estimation error with $E_t[s_t - \hat{s}_t] = 0$ and $\text{var}(s_t - \hat{s}_t) = \frac{1 - \theta}{1 - (1 - \theta)(1 + r)^2} \omega_{\zeta}^2$.

3. General Equilibrium under RI

3.1. Optimal Consumption and Savings functions

Following the standard procedure in the literature, the consumption function and the value function under RI can be obtained by solving the following stochastic Bellman equation:

$$\hat{v}(\hat{s}_t) = \max_{c_t} \left\{ -\frac{1}{\alpha} \exp(-\alpha c_t) + \frac{1}{1 + \rho} E_t[\hat{v}(\hat{s}_{t+1})] \right\}, \quad (16)$$

⁹One could instead model RI as having a fixed marginal cost of acquiring channel capacity; as discussed in Luo and Young (2014) the two formulations are equivalent for the questions at hand here. See Section 3.3 for a detailed discussion on this case.

subject to (13)-(15). The following proposition summarizes the main results from the above precautionary-savings model with RI:

Proposition 1. *For a given Kalman gain, θ , the value function is*

$$\widehat{v}(\widehat{s}_t) = -\frac{1}{r\alpha} \exp \left(-r\alpha \left\{ \widehat{s}_t - \frac{1}{r\alpha} \ln(1+r) + \frac{1}{r^2\alpha} \left[\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t \left[\exp \left(-r\alpha \widehat{\zeta}_{t+1} \right) \right] \right] \right\} \right), \quad (17)$$

the consumption function is

$$c_t^* = r \left\{ \widehat{s}_t + \frac{1}{\alpha r^2} \left[\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t \left[\exp \left(-r\alpha \widehat{\zeta}_{t+1} \right) \right] \right] \right\}, \quad (18)$$

and the saving function is

$$d_t^* = (1 - \phi_1) \phi (y_t - \bar{y}) + r (s_t - \widehat{s}_t) + \frac{1}{r\alpha} \left[\ln E_t \left[\exp \left(-r\alpha \widehat{\zeta}_{t+1} \right) \right] - \Psi(r) \right], \quad (19)$$

where $s_t - \widehat{s}_t$ is an $MA(\infty)$ estimation error process given in (15) and $\Psi(r) = \ln \left(\frac{1+\rho}{1+r} \right)$.

Proof. See Appendix 6.1. ■

Comparing (4) with (18), it is clear that the two consumption functions are identical except that we replace s_t with \widehat{s}_t and $\zeta_{t+1} (\equiv \phi w_{t+1})$ with $\widehat{\zeta}_{t+1}$, respectively. First, given that

$$\begin{aligned} \ln E_t [\exp (-r\alpha \zeta_{t+1})] &= \frac{1}{2} (r\alpha)^2 \omega_\zeta^2, \\ \ln E_t [\exp (-r\alpha \widehat{\zeta}_{t+1})] &= \frac{1}{2} \Gamma(\theta, r) (r\alpha)^2 \omega_\zeta^2, \end{aligned}$$

it is straightforward to show that the precautionary saving premium due to limited attention is

$$P_{ri} \equiv \frac{1}{\alpha r} \ln E_t \left[\exp \left(-r\alpha \widehat{\zeta}_{t+1} \right) - \exp \left(-r\alpha \zeta_{t+1} \right) \right] = \frac{1}{2} (\Gamma(\theta, r) - 1) r\alpha \omega_\zeta^2, \quad (20)$$

which is clearly decreasing with the degree of attention (θ), and is increasing with the coefficient of absolute risk aversion (α) and the persistence and volatility of the income shock (ϕ and σ) for any given θ . In other words, RI can amplify the impact of the interaction of risk aversion and income uncertainty on increasing the amount of precautionary savings.

To further explore the precautionary savings premium in (20), we isolate the effects of RI on individual consumption and saving by rewriting (18) as

$$c_t^* = r\hat{s}_t + \frac{1}{r\alpha} \left\{ \Psi(r) - \left[\ln E_t [\exp(-r\alpha\theta(1+r)(s_t - \hat{s}_t))] + \frac{1}{2} (r\alpha\theta\omega_\zeta)^2 + \frac{1}{2} (1-\theta) \Gamma(\theta, r) (r\alpha\omega_\zeta)^2 \right] \right\}, \quad (21)$$

where $\Psi(r) = \ln\left(\frac{1+\rho}{1+r}\right)$ measures the relative importance of impatience to the interest rate in determining optimal consumption (it is greater than 0 if $\rho > r$),

$$\frac{1}{\alpha r} \ln E_t [\exp(-\alpha r\theta(1+r)(s_t - \hat{s}_t))] = r\alpha\theta(1-\theta) \Gamma(\theta, r) (1+r)^2 \omega_\zeta^2$$

is the precautionary savings premium due to the time t estimation error, $(r\alpha\theta\omega_\zeta)^2/2$ is the precautionary savings premium driven by the exogenous fundamental income shocks $\{w_t\}$, and $(1-\theta) \Gamma(\theta) (r\alpha\omega_\zeta)^2/2$ captures the precautionary savings premium driven by the endogenous noise shocks, $\{\xi_t\}$.¹⁰ Note that when θ converges to 1, the consumption function with RI, (18), reduces to that of the Wang (2003) model, (4). From (18), for finite capacity ($\kappa < \infty$ or $\theta \in (0, 1)$), the precautionary saving premium due to fundamental shocks is lower than that in the full-information case, i.e., $(r\alpha\theta\omega_\zeta)^2/2 < (r\alpha\omega_\zeta)^2/2$ because of the incomplete adjustment of consumption to the fundamental shock, while we have two new positive terms that increase the total savings more than the absolute value of the reduced savings: (i) the premium due to the estimation error and (ii) the premium due to the RI-induced endogenous noise.

Note that given the time t available information, the conditional mean of (19) can be written as:¹¹

$$\tilde{d}_t = f_t + \left(\frac{1}{2} r\alpha \Gamma(\theta, r) \omega_\zeta^2 - \frac{1}{r\alpha} \Psi(r) \right), \quad (22)$$

where the first term $f_t = (1 - \phi_1) \phi(y_t - \bar{y})$ captures the consumer's demand for savings "for a rainy day", and the second term, $\frac{1}{2} r\alpha \Gamma(\theta, r) \omega_\zeta^2$, is the certainty equivalent of the innovation to the perceived state, \hat{s}_t .

3.2. Existence of General Equilibrium

As in Wang (2003), we assume that the economy is populated by a continuum of *ex ante* identical, but *ex post* heterogeneous agents, of total mass normalized to one, with each agent solving the

¹⁰This result is derived by using Equation (15) and the iid property of the processes $\{\hat{\zeta}_t\}$, $\{\zeta_t\}$, and $\{\xi_t\}$.

¹¹Note that here we use the fact that $E_t[s_t - \hat{s}_t] = 0$.

optimal consumption and savings problem with RI specified in (16). Similar to Huggett (1993), the risk-free asset in our model is a pure-consumption loan and is in zero net supply. The initial cross-sectional distribution of permanent income is assumed to be its stationary distribution $\Phi(\cdot)$. By the law of large numbers in Sun (2006), provided that the spaces of agents and the probability space are constructed appropriately, aggregate permanent income and the cross-sectional distribution of permanent income $\Phi(\cdot)$ are constant over time.

Proposition 2. *The total savings demand “for a rainy day” in the precautionary savings model with RI equals zero for any positive interest rate. That is, $F_t(r) = \int_{y_t} f_t(r) d\Phi(y_t) = 0$, for $r > 0$.*

Proof. The proof uses the LLN and is the same as that in Wang (2003). ■

Proposition 2 states that the total savings “for a rainy day” is zero, at any positive interest rate. Therefore, from (20), for $r > 0$, the expression for total savings under RI in the economy at time t is

$$D(\theta, r) \equiv \frac{1}{r\alpha} (\Pi(\theta, r) - \Psi(r)) = \frac{1}{r\alpha} \left[\frac{1}{2} (r\alpha)^2 \Gamma(\theta, r) \omega_\zeta^2 - \Psi(r) \right], \quad (23)$$

where $\Pi(\theta, r) = \frac{1}{2} (r\alpha)^2 \Gamma(\theta, r) \omega_\zeta^2$ measures the amount of precautionary savings, and $\Psi(r)$ captures the dissaving effects of impatience. Given (23), an equilibrium under RI is defined by an interest rate r^* satisfying

$$D(\theta, r^*) = 0. \quad (24)$$

The following proposition shows the existence of the equilibrium and the PIH holds in the RI general equilibrium:

Proposition 3. *There exists at least one equilibrium with an interest rate $r^* \in (0, \rho)$ in the precautionary-savings model with RI. In any such equilibrium, each agent’s consumption is described by the PIH, in that*

$$c_t^* = r^* \hat{s}_t, \quad (25)$$

where $\hat{s}_t = E[s_t | \mathcal{I}_t]$ is the perceived value of permanent income. Furthermore, in this equilibrium, the evolution equations of wealth and consumption are

$$\Delta a_{t+1}^* = \frac{1 - \phi_1}{1 + r^* - \phi_1} (y_t - \bar{y}) + r^* (s_t - \hat{s}_t), \quad (26)$$

$$\Delta c_{t+1}^* = r^* \hat{\zeta}_{t+1}, \quad (27)$$

respectively, where $\widehat{\zeta}_{t+1}$ is specified in (14) with $E_t [\widehat{\zeta}_{t+1}] = 0$, $\text{var} (\widehat{\zeta}_{t+1}) = \Gamma (\theta, r^*) \omega_{\zeta}^2$, and $\Gamma (\theta, r^*) = \frac{\theta}{1-(1-\theta)(1+r^*)^2}$.

Proof. If $r > \rho$, the two terms, $\Pi (\theta, r)$ and $\Psi (r)$, in the expression for total savings $D (\theta, r^*)$, are positive, which contradicts the equilibrium condition, $D (\theta, r^*) = 0$. Since $\Pi (\theta, r) - \Psi (r) < 0 (> 0)$ when $r = 0$ ($r = \rho$), the continuity of the expression for total savings implies that there exists at least one interest rate $r^* \in (0, \rho)$ such that $D (\theta, r^*) = 0$. From (18), we can obtain the individual's optimal consumption rule under RI in general equilibrium as $c_t^* = r^* \widehat{s}_t$. Substituting (25) into (2) yields (26). Using (13) and (25), we can obtain (27). ■

The intuition behind Proposition 3 is similar to that in Wang (2003). With an individual's constant total precautionary savings demand $\Pi (\theta, r)$, for any $r > 0$, the equilibrium interest rate r^* must be at a level with the property that individual's dissavings demand due to impatience is exactly balanced by their total precautionary-savings demand, $\Pi (\theta, r^*) = \Psi (r^*)$. Figure 1 shows that the aggregate saving function is increasing with the interest rate, and there exists a unique interest rate r^* for every given θ such that $D (\theta, r^*) = 0$. Here we choose the parameter values as follows: $\alpha = 2.5$, $\phi_1 = 0.88$, $\sigma = 0.29$, and $\rho = 0.04$.¹²

Given (18) and (24), it is clear that even though the individual increases their total precautionary savings for information frictions, the level of aggregate savings also equals zero. That is, RI does not affect the aggregate wealth in the economy. In contrast, RI affects the equilibrium interest rate. With lower Shannon channel capacity, the equilibrium interest rate is lower.

From the equilibrium condition,

$$\frac{1}{2} (r^* \alpha)^2 \Gamma (\theta, r^*) \omega_{\zeta}^2 - \ln \left(\frac{1+\rho}{1+r^*} \right) = 0, \quad (28)$$

it is straightforward to show that

$$\frac{dr^*}{d\theta} = \frac{r^* (2+r^*)}{\theta [1-(1-\theta)(1+r^*)^2]} \left\{ \frac{2(1-\phi_1)[1-(1-\theta)(1+r^*)]+2r^*(1-\theta)(1+r^*)}{r^*(1+r^*-\phi_1)[1-(1-\theta)(1+r^*)^2]} \right\}^{-1} + \frac{1}{(1+r^*) \ln \left(\frac{1+\rho}{1+r^*} \right)} > 0, \quad (29)$$

where $1-(1-\theta)(1+r^*)^2 > 0$ and $\ln \left(\frac{1+\rho}{1+r^*} \right) > 0$. It is clear from this expression that r^* is locally

¹²In Section 4.1, we will provide more details about how to estimate the income process using the U.S. data. The main result here is robust to the choices of these parameter values.

unique and decreasing in the degree of inattention $1 - \theta$. Figure 2 illustrates r^* as a function of θ for different values of α , and clearly shows that the general equilibrium interest rate is increasing with the degree of attention, θ .¹³ The first row of Table 1 reports the general equilibrium interest rates for different values of θ .¹⁴ We can see from the table that r^* decreases as the degree of inattention increases. For example, if θ is reduced from 1 to 0.2, r^* is reduced from 2.95 percent to 2.77 percent. One might ask what a reasonable value of θ is, and whether a drop from 1 to 0.2 is “large” in any sense. Unfortunately, it is not straightforward to answer these questions, so we simply note that 0.2 is larger than the value needed to match portfolio holdings in Luo (2010) and is therefore not obviously unreasonable.

3.3. Elastic Capacity

As argued in Sims (2010), instead of using fixed channel capacity to model finite information-processing ability, one could assume that the marginal cost of information-processing (i.e., the shadow price of information-processing capacity) is fixed. That is, the Lagrange multiplier on (9) is constant. In the univariate case, the objective of the agent with finite capacity in the filtering problem is to minimize the discounted expected mean square error (MSE), $E_t \left[\sum_{t=0}^{\infty} \beta^t (s_t - s_t^*)^2 \right]$, subject to the information-processing constraint, or

$$\min_{\{\Sigma_t\}} \left\{ \sum_{t=0}^{\infty} \beta^t \left[\Sigma_t + \lambda \ln \left(\frac{R^2 \Sigma_{t-1} + \omega_{\zeta}^2}{\Sigma_t} \right) \right] \right\},$$

where Σ_t is the conditional variance at t , λ is the Lagrange multiplier corresponding to (9). Solving this problem yields the optimal steady state conditional variance:

$$\Sigma = \frac{R^2 (1 - \beta) \tilde{\lambda} - 1 + \sqrt{\left[R^2 (1 - \beta) \tilde{\lambda} - 1 \right]^2 + 4 \tilde{\lambda} R^2}}{2 R^2} \omega_{\zeta}^2, \quad (30)$$

where $\tilde{\lambda} = \lambda / \omega_{\zeta}^2$ is the normalized shadow price of information-processing capacity. It is straightforward to show that as λ goes to 0, $\Sigma = 0$; and as λ goes to ∞ , $\Sigma = \infty$. Note that $\frac{\partial \ln \Sigma}{\partial \ln \omega_{\zeta}^2} < 1$ if we adopt the assumption that λ is fixed, while $\frac{\partial \ln \Sigma}{\partial \ln \omega_{\zeta}^2} = 1$ in the fixed κ case. Comparing (30) with (10), it is clear that the two RI modeling strategies are observationally equivalent in the sense that they

¹³Here we use the same parameter values as above. For different values of σ and ϕ_1 , we have the similar pattern of the equilibrium interest rate as in the benchmark case.

¹⁴Here we also set $\gamma = 2.5$, $\phi_1 = 0.88$, $\sigma = 0.29$, and $\rho = 0.04$.

lead to the same conditional variance if the following equality holds:

$$\kappa(R, \tilde{\lambda}) = \ln R + \frac{1}{2} \ln \left(1 + \frac{2}{R^2(1-\beta)\tilde{\lambda} - 1 + \sqrt{[R^2(1-\beta)\tilde{\lambda} - 1]^2 + 4\tilde{\lambda}R^2}} \right). \quad (31)$$

In this case, the Kalman gain is

$$\theta(R, \tilde{\lambda}) = 1 - \frac{1}{R} \left\{ 1 + \frac{2}{R^2(1-\beta)\tilde{\lambda} - 1 + \sqrt{[R^2(1-\beta)\tilde{\lambda} - 1]^2 + 4\tilde{\lambda}R^2}} \right\}^{-1}. \quad (32)$$

It is obvious that κ converges to its lower limit $\underline{\kappa} = \ln(R) \approx R - 1$ as λ goes to ∞ ; and it converges to ∞ as λ goes to 0. In other words, using this RI modeling strategy, the consumer is allowed to adjust the optimal level of capacity in such a way that the marginal cost of information-processing for the problem at hand remains constant. Note that this result is consistent with the concept of ‘elastic’ capacity proposed in Kahneman (1973).

Given this relationship between λ and θ (or κ), in the following analysis we just use the value of θ to measure the degree of attention. It is worth noting that although the above two RI modeling strategies, inelastic and elastic capacity, are observationally equivalent in the “static” sense, they have distinct implications for the model’s propagation mechanism if the economy is experiencing regime switching (e.g., before and after the great moderation). With inelastic capacity, the propagation mechanism governed by the Kalman gain is fixed regardless of changes in fundamental uncertainty, while with elastic capacity the propagation mechanism will change in response to changes in fundamental uncertainty.

The key difference between the elastic capacity case and the fixed capacity case is that κ and θ now depend on both the equilibrium interest rate and labor income uncertainty for a given λ . The equilibrium interest rate is now determined implicitly by the following function:

$$D(\theta(r^*, \tilde{\lambda}), r^*) \equiv \frac{1}{r^* \alpha} \left[\frac{1}{2} (r^* \alpha)^2 \Gamma(\theta(r^*, \tilde{\lambda}), r^*) \omega_{\xi}^2 - \ln \left(\frac{1+\rho}{1+r^*} \right) \right] = 0. \quad (33)$$

Figure 3 illustrates how r^* varies with labor income uncertainty, σ , for fixed information-processing cost, λ . It clearly shows that the aggregate saving function is increasing with the interest rate and the general equilibrium interest rate is decreasing with labor income uncertainty. We can see from

Table 2 that when the economy becomes more volatile (i.e., larger σ), the Kalman gain (θ) increases while the equilibrium interest rate (r^*) decreases. This result is different from that obtained in the fixed capacity case in which θ and r^* move in the same direction. (See Table 1.) The main reason for this result is that income uncertainty affects the equilibrium interest rate via two channel: (i) the direct channel which leads to higher aggregate savings (i.e., the ω_ξ^2 term in (33)) and (ii) the indirect channel which leads to lower aggregate savings (i.e., the $\theta(r^*, \tilde{\lambda})$ term in (33)), and the direct channel dominates.

3.4. Comparison with Habit Formation

Habit formation (HF) has been widely used in economic models to study the consumption and saving behavior and asset pricing. It can be modelled directly as a structure of preferences in which psychological factors make consumers prefer to gradual adjustment in consumption, thereby consumption volatility is more painful than it would be in the absence of habit persistence. As shown in Luo (2008) within a partial equilibrium PIH framework, the key difference between HF and RI is that individual consumption under RI reacts not only to income shocks but also to its own endogenous noises induced by finite capacity. In other words, although the two models lead to the same response of consumption to income shocks, consumption growth under RI is more volatile than that under habit formation at the individual level because the noise terms due to RI also contribute to the relative volatility of consumption. In contrast, the effects of HF and RI on aggregate consumption could be similar because aggregating across all individuals would weaken or even eliminate the impacts of the endogenous noises on consumption growth.¹⁵

In this section, we compare the different implications of HF and RI in the general equilibrium framework. Following Alessie and Lusardi (1997), we introduce HF into the FI-RE model specified in Section 2.1 by assuming that the utility function takes the following form:

$$u(c_t, c_{t-1}) = -\frac{1}{\alpha} \exp(-\alpha(c_t - \gamma c_{t-1})). \quad (34)$$

Using the same solution method used in Section 2.1, we can solve for the consumption function

¹⁵Reis (2006) showed that infrequent adjustments due to inattentiveness also leads to gradual responses of aggregate consumption to income shocks, which matches the empirical evidence. See Luo (2008) for a discussion on Sims' rational inattention hypothesis and Reis' inattentiveness hypothesis.

under HF as follows:¹⁶

$$c_t = \frac{r(1+r-\gamma)}{1+r} s_t + \frac{\gamma}{1+r} c_{t-1} + \frac{1}{r\alpha} \left(\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t \left[\exp \left(-\frac{r\alpha(1+r-\gamma)}{1+r} \zeta_{t+1} \right) \right] \right). \quad (35)$$

The corresponding saving function can thus be written as:

$$d_t = (1 - \phi_1) \phi(y_t - \bar{y}) + \frac{r^2 \gamma}{r+1} \frac{\zeta_t}{1 - \gamma \cdot L} - \frac{1}{\alpha(1-\gamma)} \left(\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t \left[\exp \left(-\frac{r\alpha(1+r-\gamma)}{1+r} \zeta_{t+1} \right) \right] \right).$$

Following the same definition of general equilibrium in our benchmark model, it is straightforward to show that there exists an equilibrium interest rate r^* such that:

$$\ln \left(\frac{1+\rho}{1+r^*} \right) - \frac{1}{2} (r^* \alpha)^2 \tilde{\Gamma}(\gamma, r^*) \omega_\zeta^2 = 0, \quad (36)$$

where $\tilde{\Gamma}(\gamma, r^*) = \left(\frac{1+r^*-\gamma}{1+r^*} \right)^2 < 1$, and

$$\frac{dr^*}{d\gamma} < 0. \quad (37)$$

That is, the stronger the habit persistence, the higher the equilibrium interest rate.¹⁷ In summary, we can conclude that although both RI and HF can lead to slow adjustments in consumption, they have opposite impacts on the equilibrium interest rate.¹⁸ RI has the potential to reduce the equilibrium interest rate, while HF has the potential to increase it. The main reason for this result is that RI introduces endogenous noises into the economy which creates an additional demand for precautionary savings and drives down the equilibrium interest rate.

4. Quantitative Results

4.1. Empirical Evidence

In order to measure the relative consumption dispersion in the data, $\frac{sd(\Delta c)}{sd(\Delta y)}$, we construct a panel data set which contains both consumption and income at the household level. We also use this panel to estimate the household income process. As panel data for consumption generally are not available, we first describe how we construct a consumption panel based on the Panel Study of

¹⁶See Appendix 6.2 for the derivation.

¹⁷In a partial equilibrium model, Alessie and Lusardi (1997) show that the stronger the habit, the less impact of income uncertainty on the precautionary saving term.

¹⁸It is worth noting that the mechanisms of RI and HF to generate slow adjustment are distinct. Under RI, slow adjustment is due to finite information processing capacity. In contrast, slow adjustment is optimal under FI because consumers are assumed to prefer to smooth not only consumption but also consumption growth.

Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX).

Following Blundell, Pistaferri, and Preston (2008), we define the household income as total household income (including wage, financial, and transfer income of head, wife, and all others in household) minus financial income (defined as the sum of annual dividend income, interest income, rental income, trust fund income, and income from royalties for the head of the household only) minus the tax liability of nonfinancial income. This tax liability is defined as the total tax liability multiplied by the nonfinancial share of total income. Tax liabilities after 1992 are not reported in the PSID and so we estimate them using the TAXSIM program from the National Bureau of Economic Research. Our final household income measure can be expressed as:

$$\text{income measure} = (\text{total HH income} - \text{financial income}) - \text{taxes} \times \frac{\text{total HH income} - \text{financial income}}{\text{total HH income}}.$$

The PSID does not include enough consumption expenditure data to create full picture of household nondurable consumption. Such detailed expenditures are found, though, in the CEX from the Bureau of Labor Statistics. But households in this study are only interviewed for four consecutive quarters and thus do not form a panel. To create a panel of consumption to match the PSID income measures, we use an estimated demand function for imputing nondurable consumption created by Guvenen and Smith (2014). Using an IV regression, they estimate a demand function for nondurable consumption that fits the detailed data in the CEX. The demand function uses demographic information and food consumption which can be found in both the CEX and PSID. Thus, we use this demand function of food consumption and demographic information (including age, family size, inflation measures, race, and education) to estimate nondurable consumption for PSID households, creating a consumption panel.

Our household sample selection closely follows that of Blundell et al. (2008) as well.¹⁹ We exclude households in the PSID low-income and Latino samples. We exclude household incomes in years of family composition change, divorce or remarriage, and female headship. We also exclude incomes in years where the head or wife is under 30 or over 65, or is missing education, region, or income responses. We also exclude household incomes where non-financial income is less than \$1000, where year-over-year income change is greater than \$90,000, and where year-over-year consumption change is greater than \$50,000. Our final panel contains 7,220 unique households with

¹⁹They create a new panel series of consumption that combines information from PSID and CEX, focusing on the period when some of the largest changes in income inequality occurred. For other explanations for observed consumption and income inequality, see Krueger and Perri (2006) and Attanasio and Pavoni (2012).

54,901 yearly income responses and 50,422 imputed nondurable consumption values.²⁰

With this constructed panel of household income and consumption, Figure 5 shows the relative dispersion of consumption, defined as the ratio of the standard deviation of the consumption change to the standard deviation of the income change between 1980 and 2000. The basic pattern confirms but extends the findings in Blundell et al. (2008). Basically, the relative consumption dispersion declines in the 1980s. Based on our theoretic model, we are able to quantify the contribution of rational inattention to this decline.

In order to estimate the income process, we narrow the sample period to the years 1980 – 1996, due to the PSID survey changing to a biennial schedule after 1996. To further restrict the sample to exclude households with dramatic year-over-year income and consumption changes, we eliminate household observations in years where either income or consumption has increased more than 200% or decreased more than 80% from the previous year. Following Floden and Lindé (2001), we normalize household income measures as ratios of the mean for that year. We then exclude all household values in years in which the income is less than 10% of the mean for that year or more than ten times the mean. To eliminate possible heteroskedasticity in the income measures, we follow Floden and Lindé (2001) to regress each on a series of demographic variables to remove variation caused by differences in age and education. We next subtract these fitted values from each measure to create a panel of income residuals. We then use this panel to estimate the household income process as specified by equation (3) by running panel regressions on lagged income. The estimated values of ϕ_1 , ϕ_2 , and σ are 0.0006, 0.88, and 0.29, respectively.

4.2. Empirical Implications for the Dynamics of Consumption, Wealth, and Income

Luo (2008) examines how RI affects consumption volatility in a partial equilibrium version of the PIH model presented above. In general equilibrium, since RI affects the equilibrium interest rate via interacting with the coefficient of absolute risk aversion and income uncertainty, it will have an additional effect on consumption dynamics. Using (26) and (27), we can obtain the key stochastic properties of the joint dynamics of individual consumption, income, and saving. The following proposition summarizes the implications of RI for the relative volatility of consumption to income as well as the relative volatility of financial wealth to income:

²⁰There are more household incomes than imputed consumption values because food consumption - the main input variable in Guvenen and Smith's nondurable demand function - is not reported in the PSID for the years 1987 and 1988. Dividing the total income responses by unique households yields an average of 7 – 8 years of responses per household. These years are not necessarily consecutive as our sample selection procedure allows households to be excluded in certain years but return to the sample if they later meet the criteria once again.

Proposition 4. *Under RI, the relative volatility of individual consumption growth to income growth is*

$$\mu_{cy} \equiv \frac{sd(\Delta c_t^*)}{sd(\Delta y_t)} = \phi r^* \sqrt{\frac{(1 + \phi_1) \Gamma(\theta, r^*)}{2}}, \quad (38)$$

and the relative volatility of financial wealth to income is

$$\mu_{ay} \equiv \frac{sd(\Delta a_t^*)}{sd(\Delta y_t)} = \frac{1}{\sqrt{2}(1 + r^* - \phi_1)} \sqrt{1 - \phi_1 + \frac{(r^*)^2 (1 - \theta) (1 + \phi_1)}{1 - (1 + r^*)^2 (1 - \theta)} + \frac{2r^* (1 - \theta) (1 - \phi_1^2)}{1 - \phi_1 (1 - \theta) (1 + r^*)}}. \quad (39)$$

Proof. See Online Appendix. ■

Expression (38) shows that RI has two opposing effects on consumption volatility. The first effect is direct through its presence in the expression of $\Gamma(\theta, r^*)$, whereas the second effect is through the general equilibrium interest rate (r^*) and is thus indirect. Using the expression of $\Gamma(\theta, r^*)$, it is straightforward to show that the direct effect of RI is to increase consumption volatility. The intuition is very simple: the presence of the RI-induced noise dominates the slow adjustment of consumption in determining consumption volatility at the individual level. In contrast, the indirect effect of RI will reduce consumption volatility because it reduces the general equilibrium interest rate and $\partial \Gamma(\theta, r^*) / \partial r^* > 0$. Following the literature of precautionary savings and the estimated income process in the preceding subsection, we set $\rho = 0.04$, $\alpha = 2.5$, $\sigma = 0.29$, and $\phi_1 = 0.88$. The second to fourth rows of Table 1 reports how the interest rate and the relative volatility of consumption and wealth to income vary with θ in general equilibrium. It is clear from the second row of Table 1 that RI can significantly affect the equilibrium interest rate. For example, when θ decreases from 100% to 10%, r^* decreases from 2.95% to 2.51%. From the third row of Table 1, the relative volatility of consumption growth to income growth increases with the degree of inattention. For example, when θ decreases from 100% to 10%, μ_{cy} increases from 0.191 to 0.227. That is, the direct effect of inattention via the $\Gamma(\theta, r^*)$ term dominates its indirect general equilibrium effect via r^* . We can get the same conclusion by shutting down the general equilibrium (GE) channel, see the corresponding partial equilibrium (PE) results reported in the same table. Comparing the GE and PE results in Table 1, we can see the values of μ_{cy} are slightly lower in the GE case if the interest rate is fixed as θ decreases. In other words, the general equilibrium effect of RI tends to reduce the volatility of individual consumption in this case.²¹

²¹We cannot examine the stochastic properties of aggregate consumption dynamics because all idiosyncratic shocks (income shocks and RI-induced noise shocks) cancel out after aggregating across consumers.

Another important implication of RI in general equilibrium is that RI leads to more skewed wealth inequality measured by μ_{ay} , the relative volatility of financial wealth to labor income. From the fourth row of Table 1, we can see that when θ is reduced from 1 to 0.1, μ_{ay} increases from 1.639 to 2.145. From (26), it is clear that the main driving force behind this result is the presence of the estimation error, $s_t - \hat{s}_t$ because $\partial \text{var}(s_t - \hat{s}_t) / \partial \theta < 0$. Note that although $\partial r^* / \partial \theta > 0$, the estimation error channel dominates the general equilibrium channel and raises the wealth inequality. Therefore, RI might have the potential to increase the theoretical wealth inequality, which makes the model fit the data better.²²

Table 2 reports how elastic Kalman gain, the general equilibrium interest rate, and the relative volatility of consumption and wealth to income for different values of σ . It is clear from the second row of Table 2 that the Kalman gain increases with income volatility. For example, if $\lambda = 50$, θ is almost doubled when σ increases from 0.2 to 0.4. From the third row of Table 2, we can see that RI has significant effects on the equilibrium interest rate. For example, r^* decreases from 3.184% to 2.402% when σ increases from 0.2 to 0.4. It is worth noting that in the elastic capacity case an increase in income volatility affects the equilibrium interest rate via two channels: (i) the direct channel (the ω_{ξ}^2 term in (33)) and (ii) the indirect channel (the elastic capacity θ term in (33)). The second panel of Table 2 reports the results when we shut down the indirect channel and assume that $\theta = 1$. Comparing the third and sixth rows of Table 2, we can see that the indirect channel is more important when σ is relatively low. For example, given that $\sigma = 0.1$, r^* decreases from 3.811% to 3.528% when we switch from the FI economy to the RI economy. Furthermore, the relative volatility of consumption growth to income growth decreases with the value of σ in general equilibrium. That is, consumption becomes smoother when income becomes more volatile. For example, in the equilibrium RI economy, μ_{cy} decreases from 0.243 to 0.171 when σ is doubled from 0.2 to 0.4.²³ This theoretical result can be used to explain the empirical evidence documented in Blundell, Pistaferri, and Preston (2008) that income and consumption inequality diverged over the sampling period.

To further explore this issue in our model, we do the following exercise. First, we divide the full sample into two sub-samples (1980 – 1986 and 1987 – 1996; or 1980 – 1987 and 1988 – 1996)

²²The literature has typically found that simple models based on standard CRRA preferences and on uninsurable shocks to labor income cannot account for the observed U.S. wealth distribution. For example, Aiyagari (1994) finds considerably less wealth concentration in a model with only idiosyncratic labor earnings uncertainty. However, it is worth noting that given the CARA-Gaussian setting, the model here is not suitable to address the issue like why the top 1% or 5% rich families hold a large fraction of financial wealth in the U.S. economy.

²³It is not surprising that μ_{cy} is greater in the equilibrium RI economy than in the equilibrium FI economy because the value of θ is less than 1 in the RI case. This result is the same as that we obtained in the fixed capacity case and reported in Table 1.

and apply the same estimation procedure to re-estimate σ and ϕ_2 . (See Table 3 for the estimation results.) They show that household income becomes more volatile in late periods than earlier ones. For example, the standard deviation of y is 0.47 in the sub-sample (1980 – 1986), while it is 0.59 in the sub-sample (1987 – 1996). The average values of μ_{cy} are 0.2447 and 0.1919 in the first and second sub-samples, respectively. In the elastic capacity case, using the estimated income processes in the first sub-sample, we first use $\mu_{cy} = 0.245$ to calibrate that $\lambda = 365$. Using this calibrated value of λ , we find that $\mu_{cy} = 0.196$, which matches the empirical counterpart almost perfectly.²⁴

4.3. Welfare Losses due to RI in Equilibrium

We now turn to the welfare cost of RI – how much utility does a consumer lose if the actual consumption path he chooses under RI deviates from the first-best FI-RE path in which $\theta = 1$? To answer this question, we follow Barro (2007) and Luo and Young (2010) to compute the marginal welfare cost due to RI. The following proposition summarizes the main result:

Proposition 5. *Given the initial value of the state, \hat{s}_0 , the marginal welfare cost (mwc) due to RI is given by*

$$\text{mwc}(\theta) \equiv \frac{(\partial v(\hat{s}_0) / \partial \theta) \theta}{(\partial v(\hat{s}_0) / \partial \hat{s}_0) \hat{s}_0} = \frac{\theta}{r^* \alpha} \left[r^* \alpha + \frac{1}{(1 + r^*) \hat{s}_0} \right] \frac{dr^*}{d\theta}, \quad (40)$$

where $dr^*/d\theta$ is given in (29) and $\hat{v}(\hat{s}_0) = -\exp(-r^* \alpha \hat{s}_0 + \ln(1 + r^*)) / (r^* \alpha)$. The monthly dollar loss due to deviating from the FI-RE path ($\theta = 1$) can be written as

$$\text{\$ loss}(\theta < 1) \equiv \frac{1}{12} r^* \text{mwc}(1) (1 - \theta) \hat{s}_0. \quad (41)$$

Proof. See Appendix 6.3. Since we are interested in the deviation from the FI-RE path, $\theta = 1$ is considered as the starting point. When the household deviates from 1 to θ , the percentage change in just $(\theta - 1)$. \hat{s}_0 is the given total wealth at the starting point. Finally, we need to convert the change in the \hat{s}_0 term to monthly rates by multiplying by $r^*/12$. ■

Expression (40) gives the proportionate reduction in the initial level of the perceived state (\hat{s}_0) that compensates, at the margin, for a percentage decrease in θ (i.e., stronger degree of RI) — in the sense of preserving the same effect on welfare for a given \hat{s}_0 . To do quantitative welfare analysis we need to know the value of \hat{s}_0 . First, we set $\hat{y}_0 \equiv E[y_t] = 1$, $\phi_1 = 0.88$, and the ratio of the initial level of financial wealth (\hat{a}_0) to mean income (\hat{y}_0) equal to 5.²⁵ Second, given that $\hat{s}_0 =$

²⁴Choosing another sub-samples (1980 – 1987 and 1988 – 1996) does not change our main result.

²⁵This number varies largely for different individuals, from 2 to 20. 5 is the average wealth/income ratio in the Survey

$\hat{a}_0 + \hat{y}_0 / (1 + r^* - \phi_1) + \hat{y}_0 / r^* = [5 + 1 / (1 + r^* - \phi_1) + 1 / r^*] \hat{y}_0$, we can calculate the values of the monthly dollar loss (\$loss) for different values of θ and the corresponding values of the general equilibrium interest rate. The fifth row of Table 1 reports the welfare losses for different degrees of inattention. For example, when $\theta = 0.4$, i.e., when the household deviate from the FI-RE path by 60%, the monthly dollar loss is $6.040 \times 10^{-4} \times \hat{y}_0$, whereas it decreases to $3.025 \times 10^{-4} \times \hat{y}_0$ when $\theta = 0.8$.²⁶

Another implication of the welfare losses due to RI reported in Table 1 is that there is a general equilibrium effect of RI on the welfare loss. For example, when r is set to be 2.95% in the PE case, the monthly dollar loss is larger than that obtained in the GE case for low values of θ . For example, when $\theta = 10\%$, the welfare loss in the GE case is 3.57% lower than that in the PE case. Thus, the partial equilibrium results might overstate the welfare losses. These two results thus provide some evidence that it is reasonable for consumers to learn the true state slowly due to finite capacity because the welfare loss from deviating from the FI-RE case is trivial. In other words, although consumers could devote much more capacity to processing economic information and thus improve their consumption decisions, it may be rational for them not to do so even when information costs are negligible.

5. Concluding Remarks

In this paper we have studied how rational inattention affects the interest rate and the joint dynamics of consumption and income in a Huggett-type general equilibrium model with the CARA-Gaussian specification. Specifically, we explored how RI reduces general equilibrium interest rates via increasing individual precautionary savings and compared the general equilibrium results with the partial equilibrium results. The key implication of the model is that the general equilibrium effect reduces the welfare cost of imperfect information substantially; given that those costs are already estimated to be small, our work shows that it may easily be optimal for households to operate under substantial uncertainty even if the costs of information are negligible. Finally, in Appendix 6.4, we show that the main results obtained in the benchmark model hold when we incorporate durable expenditures into the model.

of Consumer Finances 2001. We find that changing the value of this ratio only has minor effects on the welfare implication.

²⁶The main conclusion here is robust to changes in the values of α , σ , and ϕ_1 .

6. Appendix

6.1. Deriving the Consumption and Saving Functions in the Huggett-type Model with RI

Given the consumption function (4), the original budget constraint (2) can be rewritten as

$$\begin{aligned} a_{t+1} + \phi y_{t+1} + \frac{\phi \phi_0}{r} &= (1+r) a_t + y_t - c_t + \phi (\phi_0 + \phi_1 y_t + w_{t+1}) + \frac{\phi \phi_0}{r} \\ &= (1+r) \left(a_t + \phi y_t + \frac{\phi \phi_0}{r} \right) - c_t + \zeta_{t+1}, \end{aligned}$$

where the $(t+1)$ -innovation $\zeta_{t+1} = \phi w_{t+1}$ is Gaussian innovation process with mean zero and variance $\phi^2 \sigma^2$. Denote $s_t = a_t + \phi y_t + \phi \phi_0 / r$, the new budget constraint and the consumption function can be rewritten as

$$\begin{aligned} s_{t+1} &= (1+r) s_t - c_t + \zeta_{t+1}, \\ c_t &= r \left\{ s_t + \frac{1}{r^2 \alpha} \left[\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t [\exp(-r\alpha \phi w_{t+1})] \right] \right\}. \end{aligned}$$

Under RI, the first-order condition with respect to c and the Envelope Theorem give us

$$\begin{aligned} u'(c_t) &= \frac{1}{1+\rho} E_t [\hat{v}'(\hat{s}_{t+1})], \\ \hat{v}'(\hat{s}_t) &= \frac{1+r}{1+\rho} E_t [\hat{v}'(\hat{s}_{t+1})], \end{aligned}$$

which imply that

$$u'(c_t) = \frac{1}{1+r} \hat{v}'(\hat{s}_t). \quad (42)$$

Conjecture that the value function takes the form

$$\hat{v}(\hat{s}_t) = -\frac{1}{r\alpha} \exp[-r\alpha (\hat{s}_t + b)]. \quad (43)$$

Combining the exponential utility, (42) and (43), the candidate optimal consumption is given by

$$c_t^* = r \left[\hat{s}_t + b + \frac{1}{r\alpha} \ln(1+r) \right]. \quad (44)$$

Plugging (44) into the utility function gives

$$u(c_t^*) = -\frac{1}{\alpha} \exp(-\alpha c_t^*) = -\frac{1}{\alpha} \exp\left(-\alpha r \left[\widehat{s}_t + b + \frac{1}{r\alpha} \ln(1+r)\right]\right) = \frac{1}{1+r} \widehat{v}(\widehat{s}_t). \quad (45)$$

Substituting (45) into the Bellman Equation (43) leads to

$$\widehat{v}(\widehat{s}_t) = \frac{1+r}{1+\rho} E_t[\widehat{v}(\widehat{s}_{t+1})]. \quad (46)$$

Using (43) and (12), (46) implies that

$$c_t^* = r \left\{ \widehat{s}_t + \frac{1}{r^2\alpha} \left[\ln\left(\frac{1+\rho}{1+r}\right) - \ln E_t[\exp(-r\alpha \widehat{\zeta}_{t+1})] \right] \right\}.$$

Matching coefficients in (44) and (18) gives

$$b = -\frac{1}{r\alpha} \ln(1+r) + \frac{1}{r^2\alpha} \left\{ \ln\left(\frac{1+\rho}{1+r}\right) - \ln E_t[\exp(-r\alpha \widehat{\zeta}_{t+1})] \right\}.$$

By utilizing (4), (13) and (18), we can derive the savings function as follows:

$$\begin{aligned} d_t^* &= ra_t + y_t - c_t^* \\ &= ra_t + y_t - c_t + (c_t - c_t^*) \\ &= ra_t + y_t - r \left[a_t + \phi y_t + \frac{\phi\phi_0}{r} + \frac{1}{r^2\alpha} \left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln E_t[\exp(-r\alpha\phi w_{t+1})] \right) \right] + \\ &\quad \left\{ r \left[a_t + \phi y_t + \frac{\phi\phi_0}{r} + \frac{1}{r^2\alpha} \left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln E_t[\exp(-r\alpha\phi w_{t+1})] \right) \right] \right. \\ &\quad \left. - r \left[\widehat{s}_t + \frac{1}{r^2\alpha} \left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln E_t[\exp(-r\alpha \widehat{\zeta}_{t+1})] \right) \right] \right\} \\ &= (1 - \phi_1) \phi (y_t - \bar{y}) + r(s_t - \widehat{s}_t) + \frac{1}{r\alpha} \left[\ln E_t[\exp(-r\alpha \widehat{\zeta}_{t+1})] - \ln\left(\frac{1+\rho}{1+r}\right) \right]. \end{aligned}$$

6.2. Solving the Habit Formation Model

As in Alessie and Lusardi (1997), we model habit formation in the Caballero model by assuming that the period utility is defined on $c_t - \gamma c_{t-1}$. The optimization problem for this habit formation model can be specified as follows:

$$\max_{\{c_t\}} U(c) = E_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \left[-\frac{1}{\alpha} \exp(-\alpha(c_t - \gamma c_{t-1})) \right] \right\}, \quad (47)$$

subject to the budget constraint

$$s_{t+1} = (1+r)s_t - c_t + \zeta_{t+1}, \quad (48)$$

where $s_t \equiv a_t + \phi \left(y_t + \frac{\phi_0}{r} \right)$ and $\zeta_{t+1} \equiv \phi w_{t+1} = w_{t+1} / (1+r-\phi_1)$, and $\gamma > 0$. The Bellman equation for this problem can be written as:

$$v(s_t, c_{t-1}) = \max_{c_t} \left\{ -\frac{1}{\alpha} \exp(-\alpha(c_t - \gamma c_{t-1})) + \frac{1}{1+\rho} E_t[v(s_{t+1}, c_t)] \right\}, \quad (49)$$

where $v(s, c)$ is the value function.

To solve this Bellman equation, we first conjecture that: $v(s_t, c_{t-1}) = -\exp(-a_0(s_t + a_1 c_{t-1} + a_2)) / a_0$, where a_0, a_1 , and a_2 are undetermined coefficients. The FOC w.r.t. c_t is

$$\exp(-\alpha(c_t - \gamma c_{t-1})) = \frac{1-a_1}{1+\rho} E_t[\exp(-a_0(s_{t+1} + a_1 c_t + a_2))]. \quad (50)$$

The Envelope theorem are:

$$\exp(-a_0(s_t + a_1 c_{t-1} + a_2)) = \frac{1+r}{1+\rho} E_t[\exp(-a_0(s_{t+1} + a_1 c_t + a_2))], \quad (51)$$

$$-\frac{a_1}{\gamma} \exp(-a_0(s_t + a_1 c_{t-1} + a_2)) = \exp(-\alpha(c_t - \gamma c_{t-1})). \quad (52)$$

Combining (50) and (51) yields

$$c_t = \frac{a_0}{\alpha} s_t + \left(\frac{a_0 a_1}{\alpha} + \gamma \right) c_{t-1} - \frac{1}{\alpha} \left(\ln \frac{1-a_1}{1+r} - a_0 a_2 \right). \quad (53)$$

Combining (50) and (52) yields

$$c_t = \frac{1}{a_0(1-a_1)} \left[a_0 r s_t - a_0 a_1 c_{t-1} + \ln \left(-\frac{a_1(1+\rho)}{\gamma(1-a_1)} \right) - \ln E_t(\exp(-a_0 \zeta_{t+1})) \right]. \quad (54)$$

Comparing the two consumption functions, (53) and (54), we can determine the coefficients in the conjectured value function:

$$a_0 = \frac{\alpha r (1+r-\gamma)}{1+r},$$

$$a_1 = -\frac{\gamma}{1+r-\gamma},$$

$$a_2 = \frac{1+r}{r(1+r-\gamma)} \left\{ \frac{1}{\alpha} \ln \left(\frac{1}{1+r-\gamma} \right) + \frac{1}{\alpha r} \left[\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t \left[\exp \left(-\frac{\alpha r (1+r-\gamma)}{1+r} \zeta_{t+1} \right) \right] \right] \right\}.$$

Substituting these coefficients into (53) yields the consumption function in the main text.

Substituting the state transition equation into the consumption function yields:

$$c_t - rs_t = \gamma (c_{t-1} - rs_{t-1}) - \frac{r\gamma}{r+1}\zeta_t + \frac{1}{r\alpha} \left(\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t \left[\exp \left(-\frac{\alpha r (1+r-\gamma)}{1+r} \zeta_{t+1} \right) \right] \right). \quad (55)$$

Combining (55) with $d_t \equiv ra_t + y_t - c_t$, we can rewrite the individual saving function as follows:

$$\begin{aligned} d_t &= ra_t + y_t - c_t \\ &= (1 - r / (1 + r - \phi_1)) y_t + (1 - \phi_1) \bar{y} / (1 + r - \phi_1) - r \left[+ \frac{1}{r\alpha(1-\gamma)} \left(\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t \left[\exp \left(-\frac{\alpha r (1+r-\gamma)}{1+r} \zeta_{t+1} \right) \right] \right) \right] \\ &= (1 - \phi_1) \phi (y_t - \bar{y}) + \frac{r^2\gamma}{r+1} \frac{\zeta_t}{1-\gamma \cdot L} - \frac{1}{\alpha(1-\gamma)} \left(\ln \left(\frac{1+\rho}{1+r} \right) - \ln E_t \left[\exp \left(-\frac{\alpha r (1+r-\gamma)}{1+r} \zeta_{t+1} \right) \right] \right), \end{aligned}$$

where we use the facts that $\phi_0 = (1 - \phi_1) \bar{y}$ and $\phi = 1 / (1 + r - \phi_1)$.

6.3. Computing the Welfare Loss due to RI

Given that the value function under RI in general equilibrium is

$$\hat{v}(\hat{s}_0) = -\frac{1}{r^*\alpha} \exp(-r^*\alpha\hat{s}_0 + \ln(1+r^*)), \quad (56)$$

we can compute the following partial derivatives:

$$\frac{\partial \hat{v}(\hat{s}_0)}{\partial \theta} = \frac{\exp(-r^*\alpha\hat{s}_0)}{r^{*2}\alpha} [1 + r^*\alpha\hat{s}_0(1+r^*)] \frac{dr^*}{d\theta} \text{ and } \frac{\partial \hat{v}(\hat{s}_0)}{\partial \hat{s}_0} = (1+r^*) \exp(-r^*\alpha\hat{s}_0).$$

The marginal welfare cost due to RI can thus be written as:

$$mwc \equiv \frac{(\partial v(\hat{s}_0) / \partial \theta) \theta}{(\partial v(\hat{s}_0) / \partial \hat{s}_0) \hat{s}_0} = \frac{\theta}{r^{*2}\alpha} \left[r^*\alpha + \frac{1}{(1+r^*)\hat{s}_0} \right] \frac{dr^*}{d\theta},$$

where we use the facts that in general equilibrium (i.e., $\ln \left(\frac{1+\rho}{1+r} \right) = \ln E_t \left[\exp \left(-r^*\alpha\hat{\zeta}_{t+1} \right) \right]$), and $dr^*/d\theta$ is given in (29).

6.4. An Extension to Incorporate Durable Consumption

Following Bernanke (1985) and Galí (1993), we consider an FI-RE version of the PIH model which includes both durable and nondurable consumption. The optimizing decisions of a representative

consumer in the RE-PIH model with durables goods can be formulated as

$$\max_{\{c_t, k_{t+1}\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, k_t) \right], \quad (57)$$

subject to the budget constraint

$$a_{t+1} = (1 + r) a_t + y_t - c_t - e_t, \quad (58)$$

and the accumulation equation for durables

$$k_{t+1} = (1 - \delta) k_t + e_t, \quad (59)$$

where $u(c_t, k_t) = -\exp(-\alpha_c c_t) / \alpha_c - \varrho \exp(-\alpha_k k_t) / \alpha_k$ is the utility function, c_t is consumption of nondurables, k_t is the stock of durables goods, e_t is the purchase of durable goods, $\beta = 1 / (1 + \rho)$ is the discount factor, $R = 1 + r$ is the constant gross interest rate, δ is the depreciation rate of durable goods, $\alpha_c > 0$, $\alpha_k > 0$, and $\varrho > 0$.

To incorporate RI, following the same procedure used in the our benchmark model and Luo, Nie, and Young (2014), we define a new state variable (s) as:

$$s_t \equiv a_t + \frac{1 - \delta}{R} k_t + \frac{1}{R - \phi_1} \left(\frac{\phi_0}{R - 1} + y_t \right), \quad (60)$$

which is governed by the following evolution equation:

$$s_{t+1} = R s_t - c_t - \frac{R + \delta - 1}{R} k_{t+1} + \zeta_{t+1},$$

where $\zeta_{t+1} = \phi w_{t+1} = \frac{1}{R - \phi_1} w_{t+1}$ is the innovation to s_{t+1} .

Following Luo, Nie, and Young (2014), we formulate the optimization problem for the typical household under RI:

$$v(\hat{s}_t) = \max_{\{c_t, k_{t+1}\}} E_t [u(c_t, k_t) + \beta v(\hat{s}_{t+1})]$$

subject to

$$\hat{s}_{t+1} = R \hat{s}_t - c_t - \frac{R + \delta - 1}{R} k_{t+1} + \hat{\zeta}_{t+1},$$

where $\hat{\zeta}_{t+1}$ is defined in (14) and \hat{s}_0 is given, The following proposition summarizes the results from the above dynamic program:

Proposition 6. Under RI, the functions of nondurable consumption and the stock of durable accumulation are:

$$c_t = H_c \hat{s}_t + \Omega_c + \hat{\Pi}_c, \quad (61)$$

$$k_{t+1} = \Omega + \frac{\alpha_c}{\alpha_k} c_t, \quad (62)$$

respectively, where $H_c = (R - 1) \left(1 + \frac{R+\delta-1}{R} \frac{\alpha_c}{\alpha_k}\right)^{-1}$, $\Omega = -\frac{1}{\alpha_k} \ln \left(\frac{R+\delta-1}{\varrho}\right)$, $\Omega_c = -\left(1 - \frac{1-\delta}{R}\right) \left(1 + \frac{R+\delta-1}{R} \frac{\alpha_c}{\alpha_k}\right)^{-1} \Omega$, $\hat{\Pi}_c = -\hat{\Pi} / (R - 1)$, and

$$\hat{\Pi} \equiv \frac{1}{\alpha_c} \ln(\beta R) + \frac{\alpha_c}{2} \left(\frac{R-1}{R-\phi_1}\right)^2 \left(1 + \frac{R+\delta-1}{R} \frac{\alpha_c}{\alpha_k}\right)^{-2} \Gamma(r, \theta) \phi^2 \sigma^2. \quad (63)$$

Proof. See Online Appendix. ■

Given the original budget constraint and the decision rules, the expression for individual saving, $d_t (\equiv (R - 1) a_t + y_t - c_t - [k_{t+1} - (1 - \delta) k_t])$, can be written as:

$$\begin{aligned} d_t = & \frac{R-1}{R-\phi_1} \left[1 - \left(1 + \frac{\alpha_c}{\alpha_k}\right) \left(1 + \frac{R+\delta-1}{R} \frac{\alpha_c}{\alpha_k}\right)^{-1} \right] \hat{\zeta}_t + (R-1) (s_t - \hat{s}_t) \\ & + \left[\frac{R}{H_c} - \left(1 + \frac{\alpha_c}{\alpha_k}\right) \right] \hat{\Pi} + \frac{R-1}{R-\phi_1} (y_t - \bar{y}), \end{aligned} \quad (64)$$

where $s_t - \hat{s}_t$ is defined in (15), respectively.

After aggregating across all consumers using the same law of large number we applied in the benchmark model, all the idiosyncratic shocks (including the fundamental income shocks and endogenous shocks due to RI) are canceled out and we obtain the following expression for the perceived aggregate saving:

$$D(\theta, r) \equiv \left[\frac{R}{H_c} - \left(1 + \frac{\alpha_c}{\alpha_k}\right) \right] \hat{\Pi}, \quad (65)$$

where $\hat{\Pi}$ is defined in (63). The following proposition shows the existence of the general equilibrium and the PIH holds in such an equilibrium:

Proposition 7. There exists at least one equilibrium with an interest rate $r^* \in (0, \rho)$ in the RI precautionary-savings model with durables. In any such equilibrium, the aggregate saving is zero:

$$\frac{1}{2} (\alpha_c r^*)^2 \left(1 + \frac{\delta + r^*}{1 + r^*} \frac{\alpha_c}{\alpha_k}\right)^{-2} \Gamma(r^*, \theta) \omega_\xi^2 - \ln \left(\frac{1 + \rho}{1 + r^*}\right) = 0.$$

the PIH still holds since consumption follows

$$c_t = H_c \hat{s}_t + \Omega_c.$$

Proof. The proof is the same as that in the benchmark model. Here we need to use the condition that $\frac{R}{H_c} - \left(1 + \frac{\alpha_c}{\alpha_k}\right) \neq 0$.²⁷ ■

We now examine how rational inattention affects the equilibrium interest rate in the CARA-Gaussian setting with durable consumption. Luo, Nie and Young (2014) show that if $\frac{\alpha_c}{\alpha_k} = \frac{R+\delta-1}{\varrho}$ holds, then the model here is observational equivalent to the LQ Gaussian model with durables. Using the estimated parameters in Bernanke (1985) ($\delta = 0.025$, and $\varrho = 0.0286$) and $\alpha_c = 2$ used in Caballero (1991) and Wang (2000), we calibrate the CARA parameter on durable consumption: $\alpha_k = 1.63$. Given the following parameter values: $\alpha_c = 2$, $\alpha_k = 1.63$, $\sigma = 0.29$, $\phi_1 = 0.88$, $\rho = 0.04$, $\delta = 0.025$, and $\varrho = 0.0286$, Figure 4 illustrates how r^* varies with the value of θ . The figure also clearly shows that the aggregate saving function is increasing with the interest rate and the general equilibrium interest rate is decreasing with the degree of inattention, which is consistent with the conclusion obtained in our benchmark model without durable goods.

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²⁷Note that the probability measure of $\frac{R}{H_c} - \left(1 + \frac{\alpha_c}{\alpha_k}\right) = 0$ is zero.

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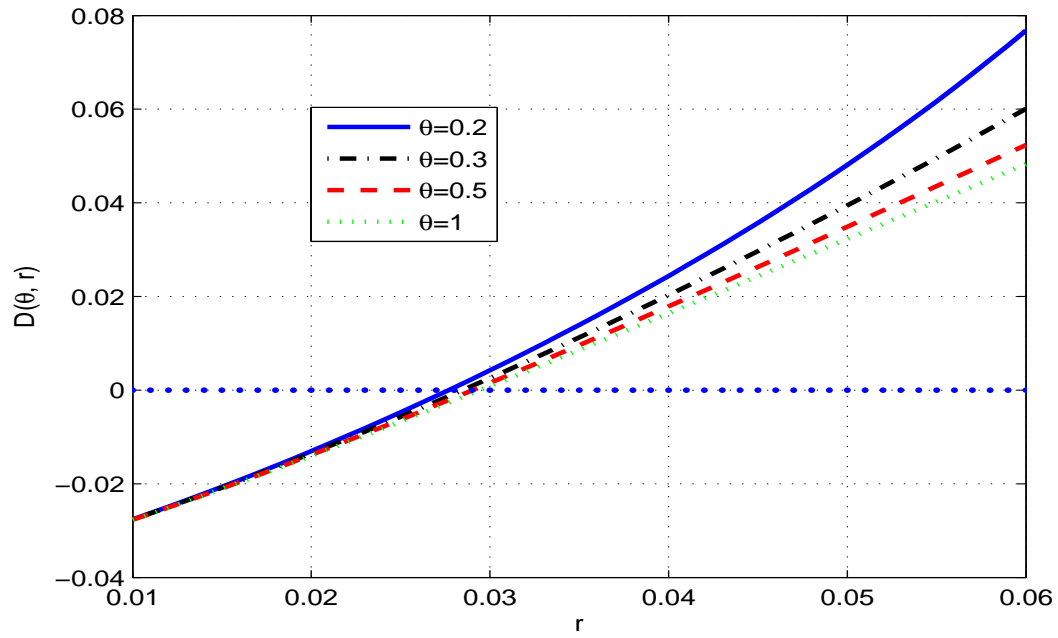


Figure 1. Effects of RI on Aggregate Saving

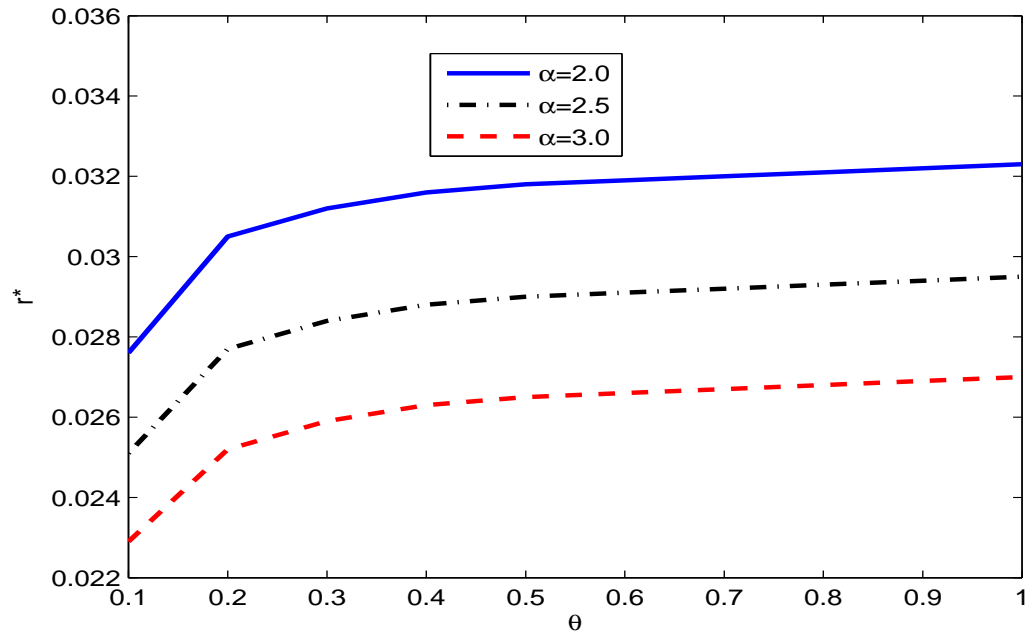


Figure 2. Effects of RI on the Equilibrium Interest Rate

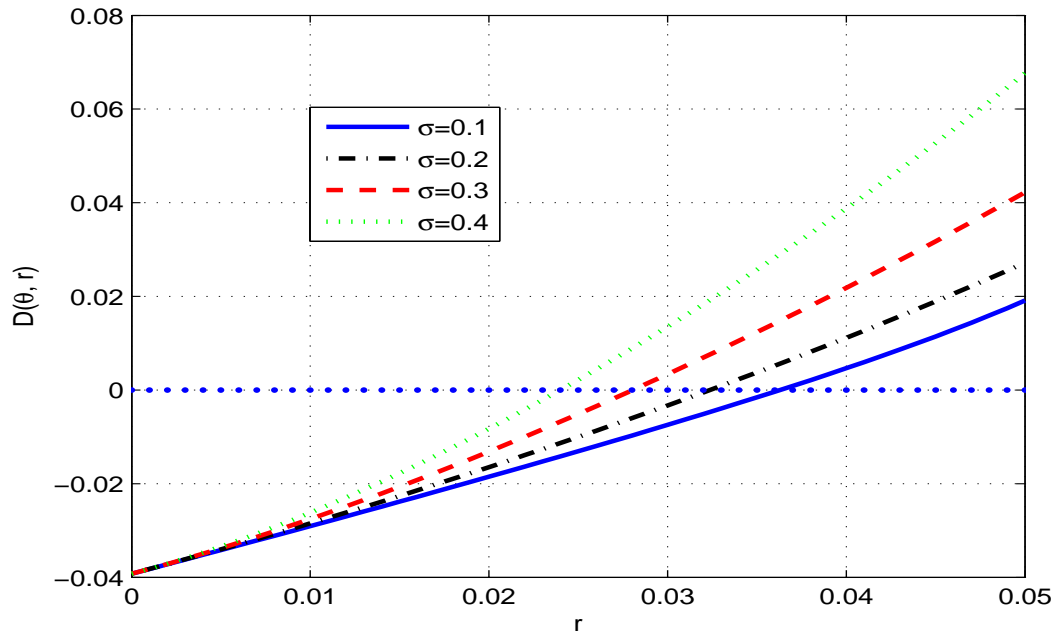


Figure 3. Effects of Income Volatility on the Interest Rate in GE (Elastic κ)

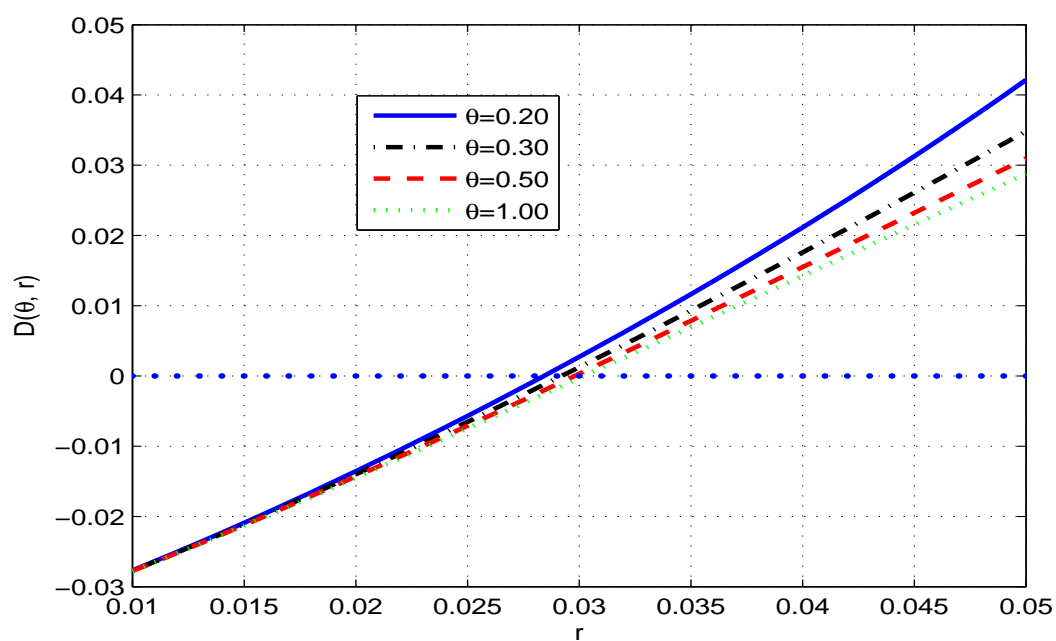
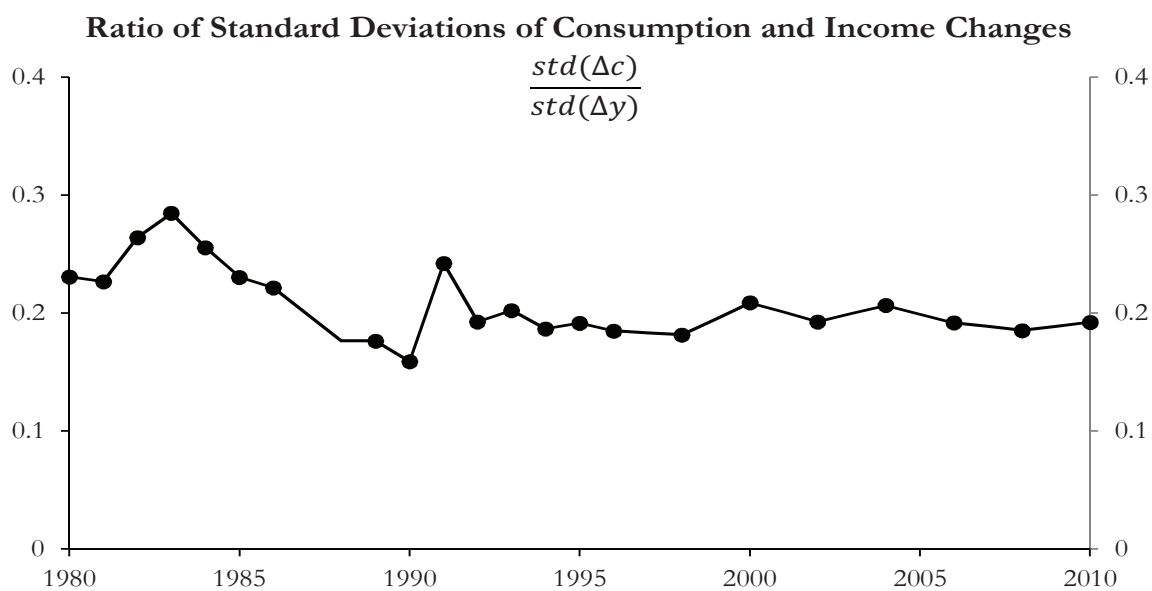


Figure 4. Effects of RI on the Equilibrium Interest Rate



*Note: Values in 1987 and 1988 are excluded due to missing PSID consumption values.

Figure 5. Relative Consumption Dispersion

Table 1. Implications of RI for interest rates, Consumption, and Welfare

	θ	0.1	0.2	0.4	0.6	0.8	1
GE	r^*	2.51%	2.77%	2.88%	2.91%	2.93%	2.95%
	μ_{cy}	0.227	0.206	0.196	0.193	0.192	0.191
	μ_{ay}	2.145	1.940	1.776	1.706	1.665	1.639
	$\$loss/\hat{y}_0$	1.316×10^{-3}	1.195×10^{-3}	9.040×10^{-4}	6.040×10^{-4}	3.025×10^{-4}	0
PE ($r = 2.95\%$)	μ_{cy}	0.282	0.219	0.201	0.195	0.193	0.191
	μ_{ay}	2.202	1.935	1.771	1.702	1.663	1.639
	$\$loss/\hat{y}_0$	1.363×10^{-3}	1.212×10^{-3}	9.089×10^{-4}	6.059×10^{-4}	3.030×10^{-4}	0

Table 2. Implications of RI in GE and PE (Elastic κ)

	$\sigma(\sigma_y)$	0.1 (0.21)	0.2 (0.42)	0.3 (0.63)	0.4 (0.84)
GE-RI ($\lambda = 50$)	θ	0.100	0.177	0.251	0.322
	r^*	3.528%	3.184%	2.771%	2.402%
	μ_{cy}	0.370	0.243	0.199	0.171
	μ_{ay}	2.318	1.973	1.880	1.846
GE-FI ($\theta = 1$)	r^*	3.811%	3.375%	2.899%	2.483%
	μ_{cy}	0.234	0.213	0.189	0.166
	μ_{ay}	1.549	1.593	1.644	1.691
PE ($r = 2.771\%$, $\lambda = 50$)	θ	0.098	0.178	0.251	0.317
	μ_{cy}	0.262	0.211	0.199	0.194
	μ_{ay}	2.187	1.968	1.880	1.828

Table 3. Estimation of the Income Process with Sub-samples

Period 1	std (ϵ_{it})	ϕ_{it}	std (y_{it})	Period 2	std (ϵ_{it})	ϕ_{it}	std (y_{it})
1980 – 1986	0.25	0.85	0.47	1987 – 1996	0.29	0.87	0.59
1980 – 1987	0.26	0.86	0.51	1988 – 1996	0.29	0.84	0.53